Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 30, 2004*
Accepted: *February 23, 2004*

A compact representation of the $\gamma\gamma ggg \to 0$ amplitude

Thomas Binoth
Institut für Theoretische Physik und Astrophysik, Universität Würzburg
Am Hubland, D-97074 Würzburg, Germany
E-mail: binoth@physik.uni-wuerzburg.de

Jean-Philippe Guillet and Farvah Mahmoudi
LAPTH
Chemin de Bellevue B.P. 110, F-74941 Annecy-le-Vieux, France
E-mail: guillet@lapp.in2p3.fr, mahmoudi@lapp.in2p3.fr

Abstract: A compact representation of the loop amplitude $\gamma\gamma ggg \to 0$ is presented. The result has been obtained by using helicity methods and sorting with respect to an irreducible function basis. We show how to convert spinor representations into a field strength representation of the amplitude. The amplitude defines a background contribution for Higgs boson searches at the LHC in the channel $H \to \gamma\gamma + jet$ which was earlier extracted indirectly from the one-loop representation of the 5-gluon amplitude.

Keywords: QCD, NLO Computations.

Contents

JHEP02(2004)057

1. Introduction

Collider experiments at the TeV scale and especially the forthcoming LHC experiment at CERN produce a large amount of data which has to be cross checked with theoretical predictions. To understand signatures from the Standard Model and beyond on a quantitative level, a good understanding of signal *and* background reactions is mandatory. This fact motivates a considerable effort from the theory side to describe prominent signal and background reactions beyond tree level. This task is highly challenging due to the combinatorial complexity of the Feynman diagrammatic approach if the number of external particles increases. Even the number of known 5-point 1-loop amplitudes is very restricted. Notable exceptions relevant for multi-jet [1]–[4] and Higgs physics exist [5]–[10]. Up to now not a single Standard Model process which has generic $2 \to 4$ kinematics is computed at the one-loop level although this is relevant for many search channels at the LHC. Examples for such amplitudes are provided only for the Yukawa model [11, 12] or for special helicity configurations [13]–[18].

To attack such a highly involved problem new methods have to be investigated and efficient, constructive algorithms have to be developed which allow an automated computation of the corresponding amplitudes. To test and investigate new ideas and concepts we calculate the $gg \to \gamma\gamma g$ amplitude in the present paper.

In hadronic collisions this amplitude is relevant for the production of photon pairs in association with a jet and as such a contribution of the background to the Higgs boson search channel $H \to \gamma\gamma + \text{jet}$. A phenomenological analysis has already been provided in [19, 20, 21]. Further this amplitude contributes to the 2-loop corrections for $gg \to \gamma\gamma$ [22]. In both articles the $gg \to \gamma\gamma g$ amplitude has been extracted from the well known 1-loop 5-gluon amplitude [1] by replacing two gluons by photons and modifying the colour algebra of the original result. No direct computation has been presented yet. We will see that a compact representation for this amplitude can be obtained by decomposing the amplitude in helicity components and by using an irreducible function basis.

In the next section 2 we will explain the organization of our calculation and the used methods. Especially we will show how spinorial representations can be retranslated into field strength representations of the result and give some rules to switch between equivalent representations. In section 3 our result will be presented in terms of helicity amplitudes which are expressed by field strength tensors. Section 4 will close the paper with a summary and an outlook for future applications of our methods.

2. Algebraic methods

We will outline here briefly the basic technology we have used to perform the calculation. We will focus here predominantly on features which we think are not standard and will not provide all details of the given calculation in this article.

2.1 Preliminaries

We consider here the one-loop amplitude for two photon three gluon scattering, $\gamma\gamma ggg \to 0$. It is defined by QED and QCD Feynman rules. The amplitude with all momenta in-going has the following kinematics:

$$\gamma(p_1, \lambda_1) + \gamma(p_2, \lambda_2) + g(p_3, \lambda_3, c_3) + \\ + g(p_4, \lambda_4, c_4) + g(p_5, \lambda_5, c_5) \to 0 \quad (2.1)$$

λ_j and c_l are helicity and the colour indices. The conversion to physical kinematics is done by crossing rules.

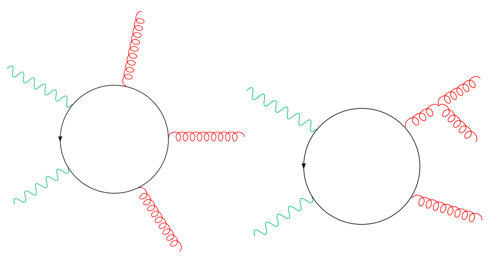

Figure 1: The two basic topologies of the $\gamma\gamma ggg$ amplitude. All the other graphs are obtained by reordering of photons and gluons.

The amplitude consists of 24 pentagon Feynman graphs and 18 box diagrams.[1] In figure 1 the two basic topologies are shown.

The colour structure of the pentagon topology is proportional to a trace of three colour matrices T^c in the fundamental representation. With the normalization $\mathrm{tr}(T^a T^b) = \delta_{ab}/2$ one has: $\mathrm{tr}(T^{c_3} T^{c_4} T^{c_5}) = (d^{c_3 c_4 c_5} + i f^{c_3 c_4 c_5})/4$. Because of Furry's theorem only the totally antisymmetric contribution in colour space remains which is also present in the 4-point topology, such that there is only one colour structure. Subsequently the amplitude

[1] The triangle diagrams with a 4-gluon vertex vanish due to colour conservation.

will be written as

$$\Gamma^{\{\lambda_j\},\{c_j\}}[\gamma\gamma ggg \to 0] = \frac{Q_q^2 g_s^3}{i\pi^2} f^{c_3 c_4 c_5} \mathcal{A}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} \,. \tag{2.2}$$

All numeric factors are shifted to $\mathcal{A}^{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5}$. In principle there are 32 helicity amplitudes but some of them are related by parity conservation. One has

$$\begin{aligned}
\mathcal{A}^{+++++}(1,2,3,4,5) &= \mathcal{A}^{-----}(1,2,3,4,5) \\
\mathcal{A}^{++++-}(1,2,3,4,5) &= \mathcal{A}^{----+}(1,2,3,4,5) \\
\mathcal{A}^{-++++}(1,2,3,4,5) &= \mathcal{A}^{+----}(1,2,3,4,5) \\
\mathcal{A}^{--+++}(1,2,3,4,5) &= \mathcal{A}^{++---}(1,2,3,4,5) \\
\mathcal{A}^{+++--}(1,2,3,4,5) &= \mathcal{A}^{---++}(1,2,3,4,5) \\
\mathcal{A}^{-+++-}(1,2,3,4,5) &= \mathcal{A}^{+---+}(1,2,3,4,5) \,.
\end{aligned} \tag{2.3}$$

All the others are obtained by interchange of the photons and/or cyclic relabeling of the gluons. Thus, it is sufficient to calculate the 6 helicity amplitudes in (2.3). Note that because of Bose symmetry and the colour structure of the amplitude anti-symmetry relations for the auxiliary objects $\mathcal{A}$ exist. For example:

$$\mathcal{A}^{+++--}(1,2,3,4,5) = -\mathcal{A}^{+++--}(1,2,3,5,4) = -\mathcal{A}^{++-+-}(1,2,4,3,5) \,. \tag{2.4}$$

The Bose symmetry allows for stringent checks of the calculation.

2.2 Spinor helicity methods

The use of helicity methods for loop calculations is well established meanwhile [23, 24]. The polarization vectors are represented by spinor products

$$\begin{aligned}
\epsilon_\mu^+(p,k) &= \frac{\langle k^-|\mu|p^-\rangle}{\sqrt{2} < k^-|p^+ >} \\
\epsilon_\mu^-(p,k) &= \frac{\langle k^+|\mu|p^+\rangle}{\sqrt{2}[pk]} \,, \qquad [pk] = \langle k^-|p^+\rangle^* \,.
\end{aligned} \tag{2.5}$$

Here k is an arbitrary light-like vector defining the transverse direction for the respective photon/gluon polarization vector.

To perform the calculation we followed two strategies. In a first approach we constructed an algebraic program where the reference vectors can be chosen arbitrarily. In a second approach we have chosen the reference momenta for the polarization vectors such that there is a global spinorial factor for all Feynman diagrams. In this way a spinorial factor can be extracted from the amplitude and all graphs can be treated uniformly for a given helicity amplitude.

To give an example for the second approach let us consider the amplitude Γ^{-+++-}. After computing the traces of the one-loop integrals the numerator of the integrals is a polynomial of scalar products of polarization vectors and four momenta, $P(\epsilon_i \cdot \epsilon_j, \epsilon_i \cdot p_j, p_i \cdot p_j)$. Products of polarization vectors are expressible in terms of scalar products between polarization vectors and momenta by using the following formulas

$$\epsilon^+(i,j)\cdot\epsilon^+(k,l) = \begin{cases} 0 & \text{if } j=l \\ \dfrac{1}{p_i\cdot p_l}\;\epsilon^+(i,j)\cdot p_l\;\;\epsilon^+(k,l)\cdot p_i & \text{if } i\neq l, j\neq l \\ \dfrac{1}{p_j\cdot p_k}\;\epsilon^+(i,j)\cdot p_k\;\;\epsilon^+(k,l)\cdot p_j & \text{if } j\neq k, i=l, j\neq l \\ -\dfrac{p_i\cdot p_j}{p_i\cdot p_m\,p_j\cdot p_m}\epsilon^+(i,j)\cdot p_k\;\;\epsilon^+(k,l)\cdot p_j & \text{if } j\neq k, i=l, j\neq l \end{cases}$$

$$\epsilon^+(i,j)\cdot\epsilon^-(k,l) = \begin{cases} 0 & \text{if } i=l \text{ or } j=k \\ \dfrac{1}{p_i\cdot p_k}\;\epsilon^+(i,j)\cdot p_k\;\;\epsilon^-(k,l)\cdot p_i & \text{if } i\neq l, j\neq k\,. \end{cases} \tag{2.6}$$

The arguments stand for the respective index of the external momentum. By applying these formulas to the numerator polynomials P, each numerator is now a linear combination of terms $\sim \prod_{j=1}^{5}\epsilon_j\cdot q_j$. Using for example the reference momenta (p_5,p_5,p_2,p_3,p_1) for the polarization vectors $\epsilon_1^-,\epsilon_2^+,\epsilon_3^+,\epsilon_4^+,\epsilon_5^-$ and formulas (2.6) one can express these products by spinor traces through

$$\begin{aligned}\prod_{j=1}^{5}\epsilon_j\cdot q_j &= \frac{\langle 5^-|q_5 1 q_1 5 q_2 2 q_3 3 q_4|4^-\rangle}{4\sqrt{2}[15]\langle 52\rangle\langle 23\rangle\langle 34\rangle[51]} \\ &= \frac{\mathrm{tr}^-(5q_5 1q_1 5q_2 2q_3 3q_4 42)}{4\sqrt{2}[15]\langle 52\rangle\langle 23\rangle\langle 34\rangle[51]\langle 42\rangle[25]}\end{aligned} \tag{2.7}$$

Here we use the notations $\mathrm{tr}^{\pm}(j,\ldots)=\mathrm{tr}^{\pm}(p_j,\ldots)=\mathrm{tr}([1\pm\gamma_5]p_j,\ldots)/2$ and $\langle j^-|l^+\rangle=\langle jl\rangle$. To achieve the trace representation a factor one, $1=\langle 4^-|2|5^-\rangle/\langle 4^-|2|5^-\rangle$, was included. Here the vector p_2 may be replaced by any other light-like vector. The spinor product is now a global factor multiplying the given helicity amplitude and does not enter the subsequent reduction algebra of scalar and tensor integrals.

After tensor and scalar integral reduction one finds a result of the form

$$\mathcal{A}^{-+++-} = \frac{A+\varepsilon(1,2,3,4)\,B}{4\sqrt{2}[15]\langle 52\rangle\langle 23\rangle\langle 34\rangle[51]\langle 42\rangle[25]} \tag{2.8}$$

A and B stand for combinations of Mandelstam variables and scalar integrals. The epsilon tensor here is defined by $\varepsilon(i,j,k,l)=\mathrm{tr}(\gamma_5\, i\, j\, k\, l)$. For the given reference momenta one has: $\epsilon_1\cdot p_5=\epsilon_2\cdot p_5=\epsilon_3\cdot p_2=\epsilon_4\cdot p_3=\epsilon_5\cdot p_1=0$. Using this fact the spinor representation can be rewritten in terms of field strength tensors, $\mathcal{F}_j^{\mu\nu}=p_j^\mu\epsilon_j^\nu-p_j^\nu\epsilon_j^\mu$. The non-abelian part of the gluon field strength tensor does not enter in the given order in α_s. By rewriting the epsilon tensor in terms of spinors and identifying the spinor products with polarization vectors and external vectors one arrives at the field strength representation of the given helicity amplitude. With the abbreviations $C_a=[A+B\,\mathrm{tr}(2354)]$, $C_b=-2s_{23}s_{24}B$ one gets:

$$\mathcal{A}_F^{-+++-}=\mathrm{Tr}(\mathcal{F}_1^-\mathcal{F}_5^-)\,\mathrm{Tr}(\mathcal{F}_2^+\mathcal{F}_3^+)\left[C_a\,p_3\cdot\mathcal{F}_4^+\cdot p_2+C_b\,p_3\cdot\mathcal{F}_4^+\cdot p_5\right]. \tag{2.9}$$

The field strength objects are more explicitly

$$\begin{aligned} \mathrm{Tr}(\mathcal{F}_i^{\pm}\mathcal{F}_j^{\pm}) &= 2\, p_i \cdot \epsilon_j^{\pm} p_j \cdot \epsilon_i^{\pm} - s_{ij}\, \epsilon_i^{\pm} \cdot \epsilon_j^{\pm} \\ p_i \cdot \mathcal{F}_j^{\pm} \cdot p_k &= \frac{(s_{ij}\, p_k \cdot \epsilon_j^{\pm} - s_{jk}\, p_i \cdot \epsilon_j^{\pm})}{2}\,. \end{aligned} \tag{2.10}$$

The Mandelstam variables are defined by $s_{ij} = 2p_i \cdot p_j$. The obtained representations are not unique and we will discuss below how to transform the result into equivalent representations. An analogous approach is possible for all the other helicity amplitudes, too. We will see below that all amplitudes are constructed from traces and so-called tails [25], which are Lorentz invariants built out of field strength tensors and external momenta. Here only the most simple type for a tail term appears, $p_j \cdot \mathcal{F}_k^{\pm} \cdot p_l$.

Using helicity methods in amplitude calculations typically leads to expressions where spinor products are the basic building blocks. Within the method presented here complex phases do not appear in the final results, which is of advantage concerning numerical evaluation.

2.3 Basic analytical structures

As explained in the last subsection each Feynman diagram is a linear combination of spinor terms $\langle i^- | \dots k \dots p_l \dots | j^- \rangle$. It was already observed elsewhere [26]–[28] that it is always possible to convert such an expression to a form with inverse propagators and a spinor trace which contains at most one power of the loop momentum. This means that only rank 1 5-point tensor integrals have to be reduced which simplifies the calculation considerably. It is easy to see that this statement holds true for a general massive 5-point amplitude too, by using the helicity formalism for massive particles and applying the same reasoning as outlined above. A similar observation was made recently in [29]. We want to remark that an equivalent statement is true for general N-point amplitudes.

Note that by power counting all 5-point graphs are IR and UV finite. The tensor reduction introduces spurious UV and IR divergences. The box topologies are UV divergent but the divergence cancels in the sum over all box graphs. For the reduction of the rank 1 5-point function and the 4-point tensor integrals we applied the formalism outlined in [31, 32]. After tensor reduction one is left with scalar integrals which are reduced to a basic set of integrals using well known formulas [31, 33]. We have used FORM [30] to perform the tensor and scalar integral reductions.

The 5-point functions are reduced to box integrals with one external massive line. The latter are reduced to triangle functions and box functions in $(n+2)$ dimensions, where $n = 4 - 2\epsilon$. In this way the spurious infrared divergences are isolated in a transparent way. For a given ordering of the external legs, say (1,2,3,4,5), one finds the following basis of scalar functions, see also figure 2:

$$\begin{aligned} &I_4^{n+2}(p_1, p_2, p_3, p_4 + p_5) + 4 \text{ cyclic permutations} \\ &I_3^{n}(p_1, p_2 + p_3, p_4 + p_5) + 4 \text{ cyclic permutations} \\ &I_3^{n}(p_1, p_2, p_3 + p_4 + p_5) + 4 \text{ cyclic permutations} \\ &I_2^{n}(p_1 + p_2, p_3 + p_4 + p_5) + 4 \text{ cyclic permutations} \end{aligned}$$

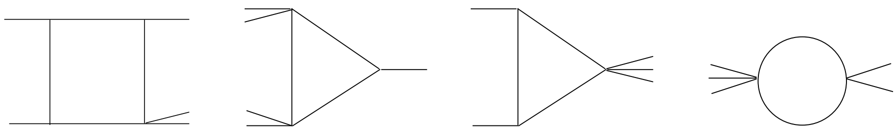

Figure 2: The scalar graph topologies of the $\gamma\gamma ggg$ amplitude.

We are following the notation in [31]. In addition the amplitude contains a finite part from the ultra-violet region of the one loop integrals. Terms of order $\mathcal{O}(n-4)$ combine with UV divergent 2-point integrals $I_2^n \sim 1/\epsilon$ to constant terms. The coefficients of these functions are first simplified on a graph by graph basis. As each graph is infrared finite the triangle graphs with one and two external legs off-shell cancel as a whole. No expansion in ϵ is performed for the scalar integrals. The different analyticity properties, or in other words, the different cut structure of each basis function does not allow any cross talking between the coefficients. It follows that each graph is expressible by 6-dimensional box integrals, 2-point functions and a constant term. For later use we define the following dimensionless function:

$$F_1(s_{j_1j_2}, s_{j_2j_3}, s_{j_4j_5}) = \frac{1}{s_{j_4j_5} - s_{j_1j_2} - s_{j_2j_3}} I_4^6(p_{j_1}, p_{j_2}, p_{j_3}, p_{j_4} + p_{j_5})\,. \tag{2.11}$$

Its analytic representation is well known and can be found in [12]. Our result will be expressed entirely in terms of this function, the scalar 2-point integral in $n = 4 - 2\epsilon$ dimensions,

$$I_2^n(s_{ij}) = \frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)^2}{\Gamma(2-2\epsilon)} \frac{(-s_{ij})^{-\epsilon}}{\epsilon} \tag{2.12}$$

and a constant term. As the amplitude is UV finite also the sum of coefficients of the 2-point functions have to cancel. For each helicity amplitude the coefficient of a given function was first simplified for each graph. Then the sum over all graphs was performed and the result again simplified using MAPLE [34]. The application of the helicity method together with the sorting of a given diagram into a function bases proved to be an efficient method for the calculation of the given amplitude. The size of the expressions was never a problem in our approach as we have split up the calculation as much as possible into irreducible building blocks.

2.4 Rules for field strength objects

We have shown above that one can convert spinor representations into field strength representations of the amplitudes. The latter are not unique and we quote two useful formulas to relate equivalent representations to each other. The first is a permutation rule:

$$\begin{aligned}\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)p_i\cdot\mathcal{F}_3^+\cdot p_j = \frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)}{s_{13}^2 s_{23}}\Big\{ &\Big[s_{23}(s_{1j}s_{3i}-s_{1i}s_{3j})+s_{13}(s_{2i}s_{3j}-s_{2j}s_{3i})\Big]p_3\cdot\mathcal{F}_2^+\cdot p_1 + \\ &+ s_{12}s_{13}\left(s_{3i}\,p_3\cdot\mathcal{F}_2^+\cdot p_j - s_{3j}\,p_3\cdot\mathcal{F}_2^+\cdot p_i\right)\Big\}\,. \end{aligned} \tag{2.13}$$

Of course the formula is true for arbitrary indices. The indices 1,2,3 were taken for convenience. In a given helicity basis it is easy to see that for a given field strength tensor only two tails, $p_i \cdot \mathcal{F} \cdot p_j$, are independent. This is directly related to the fact that there are only two independent scalar products between a polarization vector and external vectors.

The second useful identity is a flipping rule:

$$\begin{aligned}\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\,\mathrm{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+) &= \mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)\,\mathrm{Tr}(\mathcal{F}_2^+\mathcal{F}_4^+)\left(\frac{\mathrm{tr}^+(1243)}{s_{13}s_{24}}\right)^2 \\ &= \frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)}{s_{13}^2}\frac{\mathrm{Tr}(\mathcal{F}_2^+\mathcal{F}_4^+)}{s_{24}^2}[\mathrm{tr}(1243)\,\mathrm{tr}^+(1243) - s_{12}s_{13}s_{34}s_{24}]\,.\end{aligned} \tag{2.14}$$

Both identities can be verified by choosing an arbitrary set of reference momenta and the identities in eq. (2.6). By complex conjugation, $(\epsilon_j^+)^* = \epsilon_j^-$, the formulas hold also for negative helicities. As $\mathrm{Tr}(\mathcal{F}_i^+\mathcal{F}_j^-) = 0$, no flipping or permutation rules exist between different helicities.

3. Result

We present now our result by expressing each helicity amplitude through field strength tensors and coefficients for the functions $F_1, I_2^n, 1$. The Ward identities are manifest in this representation.

3.1 $\mathcal{A}^{+++++}$

For the helicity amplitude with all helicities plus, all 6-dimensional box and 2-point functions cancel. This can be understood by the fact that every amplitude is defined, up to a polynomial remainder, by the sum over all cuts [16]. Each cut corresponds to the gluing of two tree level amplitudes, in our case 2 or 3 gauge bosons attached to a fermion line. The tree level amplitude of a massless fermion and two/three gauge bosons with equal helicities vanishes. It follows that no non-polynomial function, i.e. F_1 or I_2^n, can be present in the amplitude. By choosing reference momenta in a cyclic way, p_j for ϵ_{j+1}, we find a term of the form

$$\mathcal{A}^{+++++} = \frac{1}{2\sqrt{2}\langle 51\rangle\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle}\,\mathrm{tr}^+(1,2,3,5)\frac{s_{12}}{s_{35}} \tag{3.1}$$

which can be written as

$$\mathcal{A}^{+++++} = \frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\,\mathrm{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+\mathcal{F}_5^+)}{2\,s_{34}s_{45}s_{35}}\,, \tag{3.2}$$

where $F_j^{\mu\nu} = p_j^\mu\epsilon_j^\nu - p_j^\nu\epsilon_j^\mu$ is the abelian part of the field strength tensor of the photons and gluons. In this form Ward identities are manifest. The expression is symmetric under exchange of the two photons and antisymmetric under gluon exchange. Including the antisymmetric colour factor leads to the full $S_2 \otimes S_3$ Bose symmetry of this amplitude.

3.2 $\mathcal{A}^{-++++}$

The only allowed structure is a polynomial. We find

$$\mathcal{A}^{-++++} = \frac{\mathrm{Tr}(\mathcal{F}_2^+\mathcal{F}_3^+)\,\mathrm{Tr}(\mathcal{F}_4^+\mathcal{F}_5^+)}{s_{23}^2s_{45}^2}\left[C^{-++++}\,p_2\cdot\mathcal{F}_1^-\cdot p_4 - (4\leftrightarrow 5)\right] \tag{3.3}$$

with the coefficient

$$C^{-++++} = -\frac{s_{15}s_{12}}{s_{24}s_{35}} - \frac{s_{15}}{s_{35}} + \frac{s_{23}}{s_{24}} - \frac{s_{15}}{s_{34}}\,. \tag{3.4}$$

Using the shift rules one finds that $\mathcal{A}^{-++++}$ is totally anti-symmetric in the gluon indices 3,4,5. Taking into account the colour factor this leads to the full S_3 Bose symmetry of the corresponding helicity amplitude.

3.3 $\mathcal{A}^{++++-}$

Again no nontrivial function can be present due to cutting rules. For the polynomial term we find the following result

$$\mathcal{A}^{++++-} = \frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\,\mathrm{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+)}{s_{12}^2 s_{34}^2}\left[C^{++++-}\;p_1\cdot\mathcal{F}_5^-\cdot p_3 - (3\leftrightarrow 4)\right]. \tag{3.5}$$

The coefficient is given by

$$\begin{aligned} C^{++++-} = &-\frac{s_{45}s_{13}s_{14}}{s_{35}s_{15}s_{24}} - \frac{s_{13}s_{45}}{s_{15}s_{35}} + \frac{s_{45}^2}{s_{15}s_{24}} - \frac{s_{12}^2+s_{45}^2-s_{12}s_{45}}{s_{35}s_{15}} + \frac{s_{13}s_{15}}{s_{23}s_{45}} + \frac{s_{13}-s_{34}}{s_{23}} - \\ &-\frac{s_{34}s_{45}}{s_{23}s_{15}} + \frac{s_{15}-s_{25}}{s_{45}} - \frac{s_{23}+s_{35}}{s_{13}} - \frac{s_{23}s_{25}}{s_{13}s_{45}} + \frac{s_{34}+s_{12}}{s_{15}}\,. \end{aligned} \tag{3.6}$$

We have checked that the corresponding amplitude has a $S_2\otimes S_2$ Bose symmetry when the photons and the gluons with equal helicities are interchanged.

3.4 $\mathcal{A}^{--+++}$

Now a nontrivial cut structure is possible. The cuts which can be associated with the Mandelstam variables $s_{13}, s_{14}, s_{15}, s_{23}, s_{24}, s_{25}$ are allowed by the helicity structure. No other Mandelstam variable is to be expected as an argument of the functions F_1, I_2^n and indeed is not observed. We split the result into three pieces with indices $F, B, 1$, which belong to the part proportional to 6-dimensional boxes F_1, a part containing bubble graphs I_2^n, and a constant term, respectively.

$$\mathcal{A}^{--+++} = \mathcal{A}_F^{--+++} + \mathcal{A}_B^{--+++} + \mathcal{A}_1^{--+++}\,. \tag{3.7}$$

We find

$$\begin{aligned} \mathcal{A}_F^{--+++} &= \frac{\mathrm{Tr}(\mathcal{F}_1^-\mathcal{F}_2^-)\,\mathrm{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+)}{s_{12}^2s_{34}^2}\left[C_F^{--+++}\;p_1\cdot\mathcal{F}_5^+\cdot p_3 - (3\leftrightarrow 4)\right]F_1(s_{13},s_{14},s_{25}) - \\ &\quad - (4\leftrightarrow 5) - (5\leftrightarrow 3) + (1\leftrightarrow 2) - (1\leftrightarrow 2, 4\leftrightarrow 5) - (1\leftrightarrow 2, 5\leftrightarrow 3) \\ \mathcal{A}_B^{--+++} &= \frac{\mathrm{Tr}(\mathcal{F}_1^-\mathcal{F}_2^-)\,\mathrm{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+)}{s_{12}^2s_{34}^2}\left[C_B^{--+++}\;p_1\cdot\mathcal{F}_5^+\cdot p_3 - (3\leftrightarrow 4)\right]I_2^n(s_{15}) - \\ &\quad - (4\leftrightarrow 5) - (5\leftrightarrow 3) + (1\leftrightarrow 2) - (1\leftrightarrow 2, 4\leftrightarrow 5) - (1\leftrightarrow 2, 5\leftrightarrow 3) \\ \mathcal{A}_1^{--+++} &= \frac{\mathrm{Tr}(\mathcal{F}_1^-\mathcal{F}_2^-)\,\mathrm{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+\mathcal{F}_5^+)}{2\,s_{34}s_{45}s_{s35}}\,. \end{aligned} \tag{3.8}$$

The indicated permutations have to be applied to the respective initial term for the indices of the field strength tensors, momenta and Mandelstam variables. The coefficients are

$$
\begin{aligned}
C_F^{--+++} &= \frac{1}{2}\frac{s_{12}^2 - 2s_{13}s_{14}}{s_{35}s_{15}} - \frac{s_{14}}{s_{34}} - \frac{s_{14}}{s_{35}} \\
C_B^{--+++} &= \frac{s_{45}}{s_{15}}\left[\frac{s_{13}+s_{35}}{s_{14}+s_{45}} + \frac{s_{14}+s_{45}}{s_{13}+s_{35}}\right] + \frac{s_{45}^2 s_{13}}{s_{15}s_{35}^2} + \frac{s_{14}s_{35}}{s_{15}s_{45}} + 2\frac{(s_{15}+s_{45})^2}{s_{35}^2} - \\
&\quad - \frac{s_{13}+s_{35}}{s_{15}} - \frac{s_{14}s_{45}}{s_{15}s_{35}} - \frac{s_{45}^2}{s_{35}s_{15}} + \frac{s_{14}+s_{24}}{s_{45}} + \frac{s_{12}-s_{14}-s_{35}}{s_{14}+s_{45}} + \\
&\quad + 2\frac{s_{14}(s_{15}+s_{45})}{s_{35}^2} + \frac{s_{23}^2 s_{15}}{s_{35}^2(s_{13}+s_{35})} + \frac{2s_{45}+s_{15}}{s_{13}+s_{35}} - 2\frac{(s_{15}+s_{45})s_{23}}{s_{35}(s_{13}+s_{35})} - \\
&\quad - \frac{(2s_{45}+s_{15})}{s_{35}} + \frac{s_{13}(2s_{45}+s_{15})}{s_{35}^2}\,.
\end{aligned}
$$

In the given expressions the $S_2 \otimes S_3$ symmetry under exchange of the two photons and the three gluons is manifest after taking into account the omitted colour factor. We note that we have actually calculated all coefficients of the F_1, I_2^n functions and checked the symmetry explicitly. A further nontrivial check is provided by the fact that the sum of all coefficients of the bubble integrals add up to zero. Of course the amplitude is finite and the $1/\epsilon$ poles cancel. To show this the flipping and shifting rules (2.13), (2.14) for the field strength expressions have to be applied. The logarithmic terms can be organized as logarithms of ratios of Mandelstam variables. The presence of the constant term can be inferred from the $+++++$ amplitude. Gauge invariance predicts this term as a part of the result. The fact that no other constant term is present seems to be an effect of the high symmetry of this amplitude.

3.5 $\mathcal{A}^{+++--}$

The amplitude has a $S_2 \otimes S_2$ symmetry under exchange of the gluons with equal helicities and the photons. Although we use this symmetry in the representation we have calculated all the coefficients of the functions independently and checked the symmetries explicitly. Again we write

$$
\mathcal{A}^{+++--} = \mathcal{A}_F^{+++--} + \mathcal{A}_B^{+++--} + \mathcal{A}_1^{+++--}\,. \tag{3.9}
$$

The amplitude has less symmetries than $\mathcal{A}^{--+++}$ above:

$$
\begin{aligned}
\mathcal{A}_F^{+++--} &= \frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\,\mathrm{Tr}(\mathcal{F}_4^-\mathcal{F}_5^-)}{s_{12}^2 s_{45}^2}\times \\
&\quad \times\Big[[C_{F\,1}^{+++--}\; p_1\cdot\mathcal{F}_3^+\cdot p_4 + (1\leftrightarrow 2)]F_1(s_{14},s_{24},s_{35}) - (4\leftrightarrow 5)\Big] + \\
&\quad + \frac{\mathrm{Tr}(\mathcal{F}_4^-\mathcal{F}_5^-)}{s_{45}^2}\Bigg[\frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)}{s_{13}^2}[C_{F\,2}^{+++--}\; p_1\cdot\mathcal{F}_2^+\cdot p_4 - (1\leftrightarrow 3)]\times \\
&\qquad \times F_1(s_{14},s_{34},s_{25}) + (1\leftrightarrow 2) - (4\leftrightarrow 5) - (1\leftrightarrow 2, 4\leftrightarrow 5)\Bigg]\,.
\end{aligned}
$$

JHEP02(2004)057

The coefficients are

$$\begin{aligned}
C_{F\,1}^{+++--} &= \Big[-\frac{2s_{24}^2 s_{34}}{s_{12}^2 s_{35}} - \frac{2s_{24}(s_{23}+s_{24})}{s_{12}^2} - \frac{3s_{24}}{s_{12}} + \frac{2s_{24}(-s_{34}+2s_{24})}{s_{35}s_{12}} - \\
&\quad - \frac{(s_{14}^2+s_{24}^2)}{2s_{13}s_{34}} - \frac{(s_{14}^2+s_{24}^2)}{s_{34}s_{35}} - \frac{(2s_{14}-2s_{24})}{s_{35}} - \frac{s_{34}}{s_{35}} - \frac{(s_{34}+2s_{14})}{2s_{13}}\Big] \\
C_{F\,2}^{+++--} &= \left[\frac{s_{34}^2}{2s_{12}s_{24}} + \frac{(s_{34}+s_{45})^2}{2s_{12}s_{24}} - \frac{s_{34}}{s_{13}}\right].
\end{aligned}$$

The nonvanishing two-point functions are

$$\begin{aligned}
\mathcal{A}_B^{+++--} &= \frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\,\mathrm{Tr}(\mathcal{F}_4^-\mathcal{F}_5^-)}{s_{12}^2 s_{45}^2}\Big[[C_{B\,1}^{+++--}\; p_1\cdot\mathcal{F}_3^+\cdot p_4 + (1\leftrightarrow 2)]I_2^n(s_{34}) - (4\leftrightarrow 5)\Big] + \\
&\quad + \frac{\mathrm{Tr}(\mathcal{F}_4^-\mathcal{F}_5^-)}{s_{45}^2}\Big[\frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_3^+)}{s_{13}^2}[C_{B\,2a}^{+++--}\; p_1\cdot\mathcal{F}_2^+\cdot p_4 + C_{B\,2b}^{+++--}\; p_3\cdot\mathcal{F}_2^+\cdot p_4]\times \\
&\qquad \times I_2^n(s_{15}) + (1\leftrightarrow 2) - (4\leftrightarrow 5) - (1\leftrightarrow 2, 4\leftrightarrow 5)\Big]
\end{aligned}$$

with the coefficients

$$\begin{aligned}
C_{B\,1}^{+++--} &= \Big[-\frac{s_{12}s_{24}^2}{s_{34}(s_{23}+s_{24})^2} - \frac{s_{24}(s_{12}-s_{24})}{(s_{23}+s_{24})^2} + \frac{s_{24}(s_{45}+2s_{24}+s_{12})}{s_{34}(s_{23}+s_{24})} - \\
&\quad - \frac{2s_{24}^2}{s_{12}(s_{23}+s_{24})} + \frac{s_{12}}{(s_{23}+s_{24})} - \frac{s_{23}(3s_{14}+s_{12}+s_{23})}{s_{34}s_{13}} - \frac{s_{23}^2 s_{14}}{s_{34}s_{13}^2} + \\
&\quad + \frac{s_{23}(s_{23}+s_{12}+s_{14})}{s_{34}(s_{13}+s_{14})} - \frac{s_{23}s_{14}s_{12}}{s_{34}(s_{13}+s_{14})^2} + \frac{(s_{13}+s_{23})(-s_{23}+s_{15})}{s_{34}s_{23}} + \\
&\quad + \frac{s_{14}s_{34}(s_{12}-s_{14})^2}{s_{13}^2(s_{13}+s_{14})^2} - \frac{s_{23}s_{14}^2(-s_{14}+4s_{12})}{s_{12}s_{13}^2(s_{13}+s_{14})} - \frac{s_{23}s_{14}^2(2s_{12}-s_{14})}{s_{13}^2(s_{13}+s_{14})^2} - \\
&\quad - \frac{2s_{14}s_{25}}{s_{12}(s_{13}+s_{14})} - \frac{2s_{12}s_{34}(-2s_{14}+s_{12})}{s_{13}^2(s_{13}+s_{14})} - \frac{3s_{14}^3 s_{23}}{s_{13}^2 s_{12}(s_{13}+s_{14})} + \\
&\quad + \frac{s_{23}s_{14}(4s_{12}+5s_{14})}{s_{13}^2(s_{13}+s_{14})} - \frac{2s_{14}^2 s_{23}}{s_{13}^2 s_{12}} - \frac{2s_{23}s_{14}(s_{13}-2s_{14})}{s_{13}^2 s_{12}} + \\
&\quad + \frac{(2s_{23}+2s_{12}+2s_{34})}{s_{13}} - \frac{(s_{34}s_{14}+2s_{12}s_{34}+2s_{23}s_{14})}{s_{13}^2} - \frac{(s_{12}+s_{13}-s_{23})}{s_{23}}\Big] \\
C_{B\,2a}^{+++--} &= \Big[\frac{s_{34}^2 s_{24}^3}{(s_{23}+s_{24})^2 s_{23}s_{12}^2} - \frac{s_{34}s_{24}^2(-s_{24}+3s_{34}+2s_{13})}{s_{12}(s_{23}+s_{24})^2 s_{23}} + \\
&\quad + \frac{s_{24}(-2s_{24}+3s_{34}+2s_{13})s_{34}}{(s_{23}+s_{24})^2 s_{23}} - \frac{s_{34}(6s_{23}-3s_{34}+s_{24}-2s_{45})}{s_{12}s_{23}} + \\
&\quad + \frac{2s_{34}^2 s_{12}}{s_{23}s_{24}(s_{23}+s_{24})} - \frac{7s_{34}}{(s_{23}+s_{24})} - \frac{s_{34}s_{24}(s_{34}+3s_{13}-s_{24})}{(s_{23}+s_{24})s_{12}^2} + \\
&\quad + \frac{s_{12}s_{34}(s_{24}-s_{34})}{(s_{23}+s_{24})^2 s_{23}} - \frac{s_{13}s_{34}(s_{34}+s_{24})}{s_{23}s_{24}(s_{23}+s_{34})} - \frac{s_{45}^2}{s_{12}^2} - \\
&\quad - \frac{2s_{23}^2(3s_{34}s_{45}+s_{34}^2+3s_{45}^2)}{s_{24}s_{12}^3} - \frac{s_{34}}{(s_{23}+s_{34})} - \frac{2s_{24}s_{34}^2}{s_{12}^3} - \\
&\quad - \frac{s_{24}s_{34}^2}{s_{12}^2 s_{23}} - \frac{s_{34}(2s_{23}-3s_{34}-3s_{45}+s_{24})}{s_{12}^2} + \frac{2s_{23}(-3s_{45}^2+s_{34}^2)}{s_{12}^3} -
\end{aligned}$$

$$
\begin{aligned}
&- \frac{2s_{45}^3(2s_{23}+s_{24})}{s_{12}^3 s_{34}} + \frac{s_{34}(-2s_{45}+2s_{12}-3s_{34})}{s_{24}s_{23}} + \frac{2s_{34}^2 s_{24}^2}{(s_{23}+s_{24})s_{12}^3} - \\
&- \frac{s_{45}^3 s_{23}}{s_{12}^2 s_{34} s_{24}} + \frac{s_{34}(7s_{34}+5s_{13}+3s_{12})}{(s_{23}+s_{24})s_{24}} + \frac{5s_{34}}{s_{24}} - \frac{2s_{23}^2 s_{45}^3}{s_{12}^3 s_{34} s_{24}} - \\
&- \frac{s_{23}(4s_{34}^2+6s_{45}^2+9s_{34}s_{45})}{s_{12}^2 s_{24}} - \frac{s_{34}(6s_{34}+4s_{13}-3s_{24})}{(s_{23}+s_{24})s_{12}} + \\
&+ \left.\frac{s_{45}^2(2s_{23}+2s_{24}-s_{45}+s_{12})}{s_{12}^2 s_{34}}\right]
\end{aligned}
$$

$$
\begin{aligned}
C_{B\,2b}^{+++--} = &\left[\frac{s_{35}^2}{(s_{23}+s_{24})^2} - \frac{s_{34}(2s_{35}+s_{12})}{(s_{23}+s_{24})^2} + \frac{s_{24}(s_{24}+2s_{34}-2s_{35})}{(s_{23}+s_{24})^2} - \right.\\
&- \frac{(2s_{12}s_{34}+s_{13}^2)}{s_{23}^2} + \frac{2s_{14}s_{34}s_{23}}{(s_{23}+s_{24})s_{12}^2} - \frac{2s_{14}s_{34}s_{23}}{s_{24}s_{12}^2} - \\
&- \frac{2s_{12}s_{34}(s_{34}+s_{12})}{s_{24}s_{23}^2} - \frac{2s_{35}s_{34}(s_{24}+s_{12})}{(s_{23}+s_{34})s_{23}^2} - \frac{4s_{14}s_{23}s_{45}}{s_{24}s_{12}^2} - \\
&- \frac{2s_{14}}{(s_{23}+s_{24})} - \frac{s_{12}s_{34}(-3s_{35}+s_{12})}{s_{24}(s_{23}+s_{24})^2} - \frac{s_{12}s_{34}^2 s_{35}}{(s_{23}+s_{34})s_{23}^2 s_{24}} - \\
&- \frac{s_{14}(s_{14}+s_{24})^2}{s_{12}s_{34}s_{24}} - \frac{(2s_{13}+s_{23})}{s_{23}} + \frac{s_{34}s_{24}s_{14}}{s_{12}(s_{23}+s_{24})^2} - \\
&- \frac{s_{14}s_{12}s_{34}}{s_{24}^2(s_{23}+s_{24})} - \frac{s_{14}s_{12}s_{34}}{s_{24}s_{23}^2} + \frac{s_{14}s_{12}s_{34}}{s_{24}^2 s_{23}} - \\
&- \frac{2s_{14}s_{45}^2 s_{23}}{s_{12}^2 s_{34} s_{24}} - \frac{(2s_{14}-s_{34})s_{14}}{(s_{23}+s_{24})s_{12}} + \frac{s_{12}^2 s_{34}^2}{s_{24}^2(s_{23}+s_{24})^2} - \frac{s_{12}^2 s_{34}^2}{s_{23}^2 s_{24}^2} - \\
&- \left.\frac{s_{34}^2(s_{24}+s_{12})}{(s_{23}+s_{34})s_{23}^2} + \frac{s_{14}(3s_{14}+s_{24})}{s_{12}s_{24}} - \frac{s_{24}s_{35}^2}{(s_{23}+s_{34})s_{23}^2} - \frac{2s_{14}s_{45}^2}{s_{34}s_{12}^2}\right].
\end{aligned}
$$

The constant term is

$$
\mathcal{A}_1^{+++--} = \frac{\mathrm{Tr}(\mathcal{F}_1^+\mathcal{F}_2^+)\,\mathrm{Tr}(\mathcal{F}_4^-\mathcal{F}_5^-)}{s_{12}^2 s_{45}^2}[C_1^{+++--}\; p_2\cdot\mathcal{F}_3^+\cdot p_4 - (4\leftrightarrow 5)] \tag{3.10}
$$

with the coefficient

$$
\begin{aligned}
C_1^{+++--} = &\left[-\frac{s_{12}(s_{15}+2s_{13}+s_{14})}{s_{23}(-s_{34}+s_{15})} + \frac{(s_{15}+s_{35})(s_{23}+s_{15})s_{25}}{(-s_{34}+s_{15})s_{23}s_{34}} + \right.\\
&+ \frac{s_{14}(2s_{15}+s_{14})}{s_{13}(s_{13}+s_{14})} - \frac{(s_{13}-s_{15})^2(s_{34}+s_{13})}{(s_{13}+s_{14})s_{23}s_{14}} + \\
&+ \frac{(-s_{14}+s_{13})^2(s_{34}+s_{13})}{s_{13}s_{23}(s_{15}+s_{13})} + \frac{s_{14}(s_{35}s_{14}-s_{15}^2)}{s_{13}s_{35}(s_{15}+s_{13})} - \frac{s_{15}^2 s_{14}}{s_{13}s_{23}s_{35}} + \frac{s_{14}^2}{s_{23}s_{13}} + \\
&+ \frac{s_{23}s_{13}}{s_{14}(s_{14}-s_{35})} + \frac{s_{23}s_{15}}{s_{35}(s_{14}-s_{35})} + \frac{s_{35}(-s_{24}+2s_{23})}{s_{14}(s_{14}-s_{35})} - \frac{(s_{13}+s_{14})s_{25}}{s_{35}s_{14}} + \\
&+ \frac{(s_{13}+2s_{12})}{s_{34}} - \frac{s_{14}s_{15}(s_{13}+s_{34}-s_{15})}{s_{13}s_{35}s_{23}} + \\
&+ \frac{(s_{12}^2+2s_{12}s_{13}+s_{13}^2+s_{12}s_{25}+s_{25}s_{13}+s_{25}^2)}{s_{23}s_{34}} + \frac{(s_{14}-s_{15})}{s_{13}} -
\end{aligned}
$$

JHEP02(2004)057

$$
\begin{aligned}
&- \frac{(s_{13} - s_{34} + s_{15} - 3s_{23})}{s_{14}} + \frac{(s_{13} - s_{15})^2 (s_{34} + s_{13})}{s_{23} s_{13} s_{14}} - \\
&- \frac{(s_{15} + s_{14})(-3s_{34} + s_{14})}{s_{23} s_{13}} - \frac{(-3s_{35} + s_{13} - 3s_{15} - 2s_{14})}{s_{23}} \Bigg] .
\end{aligned}
$$

3.6 $\mathcal{A}^{-+++-}$

The amplitude has only a S_2 symmetry under exchange of the gluons with equal helicities. Again we split the amplitude into an F_1, I_2^n and constant part.

$$
\mathcal{A}^{-+++-} = \mathcal{A}_F^{-+++-} + \mathcal{A}_B^{-+++-} + \mathcal{A}_1^{-+++-} \tag{3.11}
$$

with

$$
\begin{aligned}
\mathcal{A}_F^{-+++-} = &\Bigg\{ \Bigg[[C_{F\,1}^{-+++-}\; p_1 \cdot \mathcal{F}_2^+ \cdot p_3 \; F_1(s_{13}, s_{14}, s_{25})] + \\
&+ [C_{F\,2}^{-+++-}\; p_1 \cdot \mathcal{F}_2^+ \cdot p_3 \; F_1(s_{35}, s_{45}, s_{12})] \Bigg] \frac{\mathrm{Tr}(\mathcal{F}_1^- \mathcal{F}_5^-)\, \mathrm{Tr}(\mathcal{F}_3^+ \mathcal{F}_4^+)}{s_{15}^2 s_{34}^2} + \\
&+ \Bigg[[C_{F\,3}^{-+++-}\; p_1 \cdot \mathcal{F}_4^+ \cdot p_2 + (2 \leftrightarrow 3)] \; F_1(s_{12}, s_{13}, s_{45}) + \\
&+ [C_{F\,4a}^{-+++-}\; p_1 \cdot \mathcal{F}_4^+ \cdot p_2 + C_{F\,4b}^{-+++-}\; p_1 \cdot \mathcal{F}_4^+ \cdot p_3] \; F_1(s_{25}, s_{35}, s_{14}) \Bigg] \times \\
&\times \frac{\mathrm{Tr}(\mathcal{F}_1^- \mathcal{F}_5^-)\, \mathrm{Tr}(\mathcal{F}_2^+ \mathcal{F}_3^+)}{s_{15}^2 s_{23}^2} \Bigg\} - (3 \leftrightarrow 4)
\end{aligned}
$$

and

$$
\begin{aligned}
C_{F\,1}^{-+++-} &= \frac{1}{2} \frac{s_{15}^2}{s_{12} s_{23}} - \frac{s_{13} s_{14}}{s_{12} s_{23}} - \frac{s_{14}}{s_{34}} - \frac{s_{14}}{s_{23}} \\
C_{F\,2}^{-+++-} &= -C_{F\,1}^{-+++-} \\
C_{F\,3}^{-+++-} &= -\frac{1}{2} \frac{s_{15}^2}{s_{14} s_{24}} + \frac{s_{12} s_{13}}{s_{14} s_{24}} + \frac{s_{13}}{s_{24}} + \frac{s_{13}}{s_{23}} - \frac{s_{15}^2 - 2 s_{12} s_{13}}{s_{14} s_{45}} + \\
&\quad + 2 \frac{s_{13}}{s_{23}} \left[1 + \frac{s_{24}}{s_{23}} - \frac{s_{12}}{s_{45}} - \frac{s_{12} s_{15}}{s_{23} s_{45}} \right] \\
C_{F\,4a}^{-+++-} &= \frac{1}{2} \frac{s_{15}^2 - 2 s_{12} s_{13}}{s_{14} s_{24}} - \frac{s_{13}}{s_{24}} - \frac{s_{13}}{s_{23}} \\
C_{F\,4b}^{-+++-} &= -\frac{s_{13}^2}{2 s_{14} s_{34}} - \frac{(s_{13} + s_{15})^2}{2 s_{14} s_{34}} + \frac{(s_{12} - s_{13})}{s_{34}} + \frac{s_{12}}{s_{23}} .
\end{aligned}
$$

The nonvanishing contribution of the 2-point integrals can be cast into the form

$$
\begin{aligned}
\mathcal{A}_B^{-+++-} = &\Bigg\{ \Big[C_{B\,1}^{-+++-}\; p_1 \cdot \mathcal{F}_2^+ \cdot p_3 \; I_2^n(s_{12}) + C_{B\,2}^{-+++-}\; p_1 \cdot \mathcal{F}_2^+ \cdot p_3 \; I_2^n(s_{25}) \Big] \times \\
&\times \frac{\mathrm{Tr}(\mathcal{F}_1^- \mathcal{F}_5^-)\, \mathrm{Tr}(\mathcal{F}_3^+ \mathcal{F}_4^+)}{s_{15}^2 s_{34}^2} + \\
&+ \Big[[C_{B\,3}^{-+++-}\; p_1 \cdot \mathcal{F}_4^+ \cdot p_2 + (2 \leftrightarrow 3)] \; I_2^n(s_{45}) + \\
&+ [C_{B\,4a}^{-+++-}\; p_1 \cdot \mathcal{F}_4^+ \cdot p_2 + C_{B\,4b}^{-+++-}\; p_1 \cdot \mathcal{F}_4^+ \cdot p_3)] \; I_2^n(s_{14}) \Big] \times \\
&\times \frac{\mathrm{Tr}(\mathcal{F}_1^- \mathcal{F}_5^-)\, \mathrm{Tr}(\mathcal{F}_2^+ \mathcal{F}_3^+)}{s_{15}^2 s_{23}^2} \Bigg\} - (3 \leftrightarrow 4) .
\end{aligned}
$$

Note that $I_2^n(s) \sim (-s)^{-\epsilon}/\epsilon$ leads to a logarithmic term and a pole part. The latter has to vanish because of the finiteness of the amplitude. Replacing the I_2^n integrals by the pole part only leads to a nontrivial relation between the coefficients. We have checked algebraically that the sum over the pole parts is indeed zero by using the flipping and remapping for our field strength representation rules (2.13), (2.14). Of course this provides another non-trivial check of the calculation. The coefficients of the logarithms are

$$
\begin{aligned}
C_{B\,1}^{-+++-} = \Bigg[& -\frac{2(s_{12}+s_{23})(-s_{15}+s_{13})}{s_{24}^2} - \frac{s_{45}^2(-s_{35}+s_{24})}{s_{12}(s_{14}+s_{24})^2} - \frac{s_{45}(s_{45}+2s_{24})}{s_{12}(s_{14}+s_{24})} - \\
& - \frac{2s_{14}(s_{14}-s_{24})}{s_{23}s_{45}} - \frac{s_{14}^2(-s_{14}+4s_{12})}{s_{45}s_{23}^2} + \frac{(s_{12}+s_{24})s_{24}^2}{s_{12}s_{23}^2} + \\
& + \frac{s_{14}(s_{23}+2s_{12}-2s_{24})}{s_{23}s_{45}} - \frac{s_{23}(3s_{14}-s_{23}-2s_{45})}{s_{12}s_{24}} - \frac{s_{24}(4s_{45}+s_{14}-2s_{24})}{s_{23}s_{12}} - \\
& - \frac{2s_{35}s_{24}}{s_{12}(s_{23}+s_{13})} - \frac{3s_{24}s_{35}^2}{s_{12}s_{23}^2} - \frac{(2s_{12}+s_{24})(-s_{24}+s_{13})^2}{s_{24}^2 s_{35}} - \\
& - \frac{4s_{45}s_{14}}{s_{23}s_{12}} - \frac{(s_{12}+s_{24})(-s_{24}+3s_{45}+8s_{14})}{s_{23}^2} - \frac{s_{45}^2}{s_{12}s_{24}} - \\
& - \frac{s_{35}(s_{35}-s_{23})^2 s_{45}^2}{s_{12}s_{23}^2(s_{23}+s_{13})^2} - \frac{s_{24}s_{35}^2}{(s_{23}+s_{13})^2 s_{12}} - \frac{(2s_{12}+2s_{24}-s_{35})}{(s_{23}+s_{13})} - \\
& - \frac{(-2s_{45}+s_{34})}{s_{12}} + \frac{(3s_{12}+3s_{15}+s_{23})}{s_{24}} - \frac{s_{24}s_{35}s_{45}}{s_{12}(s_{14}+s_{24})^2} + \\
& + \frac{s_{45}s_{35}^2(3s_{24}+s_{35})}{s_{12}s_{23}^2(s_{23}+s_{13})} + \frac{s_{45}^2 s_{35}}{s_{24}s_{12}(s_{14}+s_{24})} + \\
& + \frac{s_{35}s_{45}s_{24}(-s_{23}+2s_{35})}{(s_{23}+s_{13})^2 s_{12}s_{23}} + \frac{2(s_{12}s_{34}+s_{12}s_{24}-s_{45}s_{24})^3}{s_{23}^3(s_{23}+s_{13})s_{12}s_{45}} + \\
& + \frac{(s_{12}+s_{24})s_{24}(2s_{35}-3s_{45})}{s_{12}s_{23}^2} + \frac{(2s_{12}+4s_{15}+4s_{24})}{s_{23}} - \\
& - \frac{2s_{35}(s_{24}+s_{45})(s_{35}-2s_{45})}{s_{12}s_{23}(s_{23}+s_{13})} - \frac{(2s_{24}-2s_{45})}{(s_{14}+s_{24})} - \frac{2s_{23}s_{35}(s_{12}+s_{23})}{s_{12}s_{24}^2} \Bigg] \\
C_{B\,2}^{-+++-} = \Bigg[& -\frac{s_{14}(s_{14}s_{25}+s_{12}s_{24})}{(s_{24}+s_{45})s_{12}s_{24}} + \frac{s_{14}}{s_{24}} - \frac{s_{14}s_{24}s_{13}^2}{(s_{23}+s_{35})s_{23}^2 s_{12}} - \frac{s_{24}s_{35}}{s_{23}^2} + \\
& + \frac{s_{24}s_{13}^2}{s_{12}s_{23}^2} - \frac{s_{25}(s_{13}+s_{35})^2}{s_{23}^2(s_{23}+s_{35})} \Bigg] \\
C_{B\,3}^{-+++-} = \Bigg[& -\frac{s_{23}^2(s_{24}+s_{34})}{s_{14}(s_{23}+s_{13})^2} - \frac{s_{23}(s_{23}+s_{34})}{(s_{23}+s_{13})^2} - \frac{s_{12}s_{23}^2}{s_{14}s_{34}(s_{23}+s_{13})} + \\
& + \frac{s_{45}(s_{24}+s_{45})s_{23}^2}{(s_{12}+s_{23})^2 s_{14}s_{24}} - \frac{s_{45}(s_{24}+2s_{23})}{(s_{12}+s_{23})^2} - \frac{s_{12}^2 s_{34}^2}{s_{14}s_{24}^2(s_{12}+s_{23})} + \\
& + \frac{(s_{24}+s_{45})s_{23}^2}{s_{14}s_{24}(s_{12}+s_{23})} + \frac{s_{23}(2s_{34}s_{12}-s_{14}s_{23})}{s_{24}^2(s_{12}+s_{23})} - \frac{2s_{13}(s_{23}+s_{24})}{(s_{12}+s_{23})s_{23}} -
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{(s_{12}s_{24}+s_{23}^2+2s_{23}s_{24}+s_{24}^2)}{s_{14}s_{34}} + \frac{s_{34}(3s_{24}-3s_{35}+s_{23}-s_{34})}{s_{24}s_{14}} + \\
&+ \frac{(s_{24}+s_{14})}{(s_{12}+s_{23})} + \frac{(2s_{23}+2s_{34})}{s_{24}} + \frac{2s_{34}}{s_{23}} - \\
&- \frac{(s_{23}+3s_{34}-3s_{25})}{s_{14}} - \frac{(s_{23}+s_{24})}{s_{34}} - \frac{s_{34}}{(s_{23}+s_{13})}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
C_{B\,4a}^{-+++-} = &\Bigg[\frac{s_{34}s_{23}^2}{s_{14}(s_{12}+s_{24})^2} - \frac{s_{23}^3s_{14}}{s_{24}^2(s_{12}+s_{24})^2} - \frac{s_{23}(s_{35}+s_{34})(s_{24}+2s_{23})}{s_{24}(s_{12}+s_{24})^2} - \\
&- \frac{s_{23}s_{34}}{s_{35}(s_{12}+s_{24})} + \frac{(s_{35}+s_{34})}{(s_{12}+s_{24})} + \frac{s_{23}^2(3s_{13}+2s_{23})}{s_{24}^2(s_{12}+s_{24})} - \\
&- \frac{s_{35}s_{34}}{s_{14}(s_{23}+s_{35})} - \frac{(s_{34}-s_{35})}{(s_{23}+s_{35})} - \frac{3s_{34}(2s_{34}+s_{13})}{s_{24}s_{14}} - \\
&- \frac{s_{34}^2(3s_{34}-3s_{35}+2s_{13})}{s_{24}^2s_{14}} + \frac{2s_{34}^3s_{35}}{s_{14}s_{24}^3} + \frac{(s_{13}+s_{34})s_{34}}{s_{35}s_{14}} + \\
&+ \frac{s_{15}^2(s_{15}-s_{24})(2s_{14}+s_{24})}{s_{35}s_{24}^3} - \frac{3s_{15}^2}{s_{24}^2} - \frac{2s_{35}s_{23}^3}{s_{24}^3(s_{12}+s_{24})} + \\
&+ \frac{2s_{15}^2(s_{15}-3s_{34})}{s_{24}^3} - \frac{s_{24}(s_{23}+s_{35})}{s_{34}s_{14}} - \frac{(s_{34}+s_{15}-s_{24})}{s_{34}} - \frac{(s_{24}+s_{34}+s_{35})}{s_{14}}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
C_{B\,4b}^{-+++-} = &\Bigg[-\frac{s_{24}s_{23}^2}{s_{35}(s_{12}+s_{24})^2} + \frac{s_{23}(s_{23}+s_{24})}{(s_{12}+s_{24})^2} - \frac{s_{24}s_{23}^2}{s_{14}s_{35}(s_{12}+s_{24})} + \\
&+ \frac{(s_{34}-2s_{35})(s_{24}+s_{14})}{s_{34}(s_{23}+s_{35})} + \frac{(s_{23}-s_{24})(s_{23}+s_{24})}{s_{24}(s_{12}+s_{24})} + \\
&+ \frac{(s_{24}s_{35}+s_{14}s_{23}+s_{23}s_{24})^2}{s_{34}^2s_{14}(s_{23}+s_{35})} + \frac{s_{24}s_{35}}{s_{14}(s_{23}+s_{35})} + \frac{s_{23}s_{24}}{s_{14}(s_{12}+s_{24})} - \\
&- \frac{s_{15}^2(2s_{14}+s_{24})}{s_{24}^2s_{35}} + \frac{s_{24}(s_{35}+s_{24})}{s_{14}s_{34}} - \frac{s_{34}(2s_{23}-s_{34}+5s_{35})}{s_{24}s_{14}} + \\
&+ \frac{(3s_{23}+9s_{34}+6s_{24})}{s_{24}} - \frac{2s_{34}^2s_{35}}{s_{24}^2s_{14}} + \frac{4s_{34}(s_{23}+s_{34})}{s_{24}^2} + \\
&+ \frac{(3s_{24}+2s_{14})}{s_{34}} - \frac{(3s_{23}-3s_{34}+3s_{35}-2s_{24})}{s_{14}}\Bigg].
\end{aligned}
$$

Finally the constant term reads

$$
\begin{aligned}
\mathcal{A}_1^{-+++-} &= \frac{\mathrm{Tr}(\mathcal{F}_1^-\mathcal{F}_5^-)\,\mathrm{Tr}(\mathcal{F}_3^+\mathcal{F}_4^+)}{s_{15}^2s_{34}^2}[C_1^{-+++-}\ p_1\cdot\mathcal{F}_2^+\cdot p_3 - (3\leftrightarrow 4)] \\
C_1^{-+++-} &= \Bigg[-\frac{3s_{24}(2s_{24}-2s_{35}-s_{45})}{s_{23}^2} + \frac{s_{12}s_{45}}{s_{23}^2} - \frac{s_{12}(2s_{12}-3s_{35}+6s_{24})}{s_{23}^2} - \\
&\quad - \frac{(s_{23}+s_{24})s_{35}s_{23}}{s_{24}^2(s_{12}+s_{24})} - \frac{s_{45}^2s_{24}}{s_{12}s_{35}(s_{34}+s_{45})} + \frac{s_{24}(s_{23}+s_{35})^2}{s_{45}s_{12}(s_{12}+s_{23})} + \\
&\quad + \frac{s_{24}^3s_{45}}{s_{12}s_{23}^2(s_{12}+s_{23})} + \frac{s_{24}^2(s_{24}-3s_{35})}{s_{12}s_{23}(s_{12}+s_{23})} - \\
&\quad - \frac{s_{24}(3s_{45}s_{24}+3s_{45}s_{35}+2s_{35}^2)}{s_{12}s_{45}s_{23}} - \frac{2s_{35}^2s_{24}}{s_{23}(s_{23}+s_{13})s_{45}} +
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{(s_{23}+s_{24}+s_{35})}{(s_{12}+s_{23})}+\frac{s_{35}^2 s_{24}}{s_{12}s_{45}^2}+\frac{(s_{35}+2s_{13}-2s_{23})}{s_{24}}-\frac{(2s_{24}-2s_{25})}{s_{23}}+\\
&+\frac{(s_{23}+s_{24})(s_{23}-s_{24}-2s_{45})}{s_{12}(s_{12}+s_{24})}-\frac{s_{35}(s_{23}+s_{24})}{s_{23}(s_{12}+s_{24})}-\\
&-\frac{s_{45}(s_{24}-s_{45})}{s_{35}(s_{34}+s_{45})}+\frac{s_{24}(s_{24}+2s_{35})}{s_{23}(s_{12}+s_{23})}+\frac{s_{24}s_{35}}{s_{45}(s_{23}+s_{13})}+\\
&+\frac{s_{35}^2 s_{24}}{s_{45}^2(s_{23}+s_{13})}-\frac{s_{24}(s_{23}-s_{35})}{s_{12}s_{45}}+\frac{s_{35}(s_{23}-s_{35})^2}{s_{23}^2(s_{23}+s_{13})}-\\
&-\frac{s_{24}^2(2s_{24}-3s_{35}-3s_{45})}{s_{12}s_{23}^2}-\frac{s_{23}(-2s_{45}-3s_{35}+2s_{23})}{s_{24}s_{12}}+\frac{s_{23}^2 s_{35}}{s_{12}s_{24}^2}+\\
&+\frac{s_{45}(3s_{24}-s_{45})}{s_{12}s_{35}}+\frac{s_{13}(s_{13}-2s_{24})}{s_{24}s_{35}}-\\
&-\frac{(-s_{24}+s_{45}+3s_{23})}{s_{12}}-\left.\frac{s_{14}(-s_{23}+s_{14})^2}{s_{45}s_{23}^2}\right].
\end{aligned}
$$

4. Summary

We have presented a calculation of the one-loop amplitude $\gamma\gamma ggg \to 0$. By applying the spinor helicity method a decomposition in helicity amplitudes was obtained which in the end were reexpressed in terms of field strength tensors. We have derived useful relations for invariants built out of field strength tensors to switch between different field strength representations. This approach avoids complex-valued spinor products in the numerical evaluation of cross sections. Two independent algebraic programs have been designed with different implementations of the spinor helicity framework. We find agreement of both results. Further we have checked the Bose symmetries of the given helicity amplitudes. The Ward identities are manifestly fulfilled in the given representation.

The decomposition of the algebraic expressions in terms of an appropriate function basis leads to a very efficient organization of the calculation which is fully automated by using algebraic programs. The final result is compact which suggests that our method is well designed to deal with more complex amplitudes.

A phenomenological application of our calculation will be provided elsewhere.

Acknowledgments

This work was supported by the Bundesministerium für Bildung und Forschung (BMBF, Bonn, Germany) under the contract number 05HT4WWA2.

References

[1] Z. Bern, L.J. Dixon and D.A. Kosower, *One loop corrections to five gluon amplitudes*, *Phys. Rev. Lett.* **70** (1993) 2677 [hep-ph/9302280].

[2] Z. Kunszt, A. Signer and Z. Trócsányi, *One loop radiative corrections to the helicity amplitudes of QCD processes involving four quarks and one gluon*, *Phys. Lett.* **B 336** (1994) 529 [hep-ph/9405386].

[3] Z. Bern, L.J. Dixon and D.A. Kosower, *One-loop amplitudes for e^+e^- to four partons*, *Nucl. Phys.* **B 513** (1998) 3 [hep-ph/9708239].

[4] Y. Yasui, *Five-parton amplitudes with two-quark and two-photon at next-to-leading order*, *Phys. Rev.* **D 66** (2002) 094012 [hep-ph/0203163].

[5] L. Reina, S. Dawson and D. Wackeroth, *QCD corrections to associated $t\bar{t}h$ production at the Tevatron*, *Phys. Rev.* **D 65** (2002) 053017 [hep-ph/0109066].

[6] W. Beenakker et al., *NLO QCD corrections to $t\bar{t}h$ production in hadron collisions*, *Nucl. Phys.* **B 653** (2003) 151 [hep-ph/0211352].

[7] W. Beenakker et al., *Higgs radiation off top quarks at the Tevatron and the LHC*, *Phys. Rev. Lett.* **87** (2001) 201805 [hep-ph/0107081].

[8] G. Belanger et al., *Full $O(\alpha)$ electroweak corrections to double Higgs-Strahlung at the linear collider*, *Phys. Lett.* **B 576** (2003) 152 [hep-ph/0309010].

[9] G. Belanger et al., *Full one-loop electroweak radiative corrections to single Higgs production in e^+e^-*, *Phys. Lett.* **B 559** (2003) 252 [hep-ph/0212261].

[10] G. Belanger et al., *Full $O(\alpha)$ electroweak and $O(\alpha_s)$ corrections to $e^+e^- \to t\bar{t}h$*, *Phys. Lett.* **B 571** (2003) 163 [hep-ph/0307029].

[11] T. Binoth, *Progress in calculating hexagon amplitudes at one-loop*, *Nucl. Phys.* **116** *(Proc. Suppl.)* (2003) 387 [hep-ph/0211125].

[12] T. Binoth, J.P. Guillet, G. Heinrich and C. Schubert, *Calculation of 1-loop hexagon amplitudes in the Yukawa model*, *Nucl. Phys.* **B 615** (2001) 385 [hep-ph/0106243].

[13] G. Mahlon, *Multi-gluon helicity amplitudes involving a quark loop*, *Phys. Rev.* **D 49** (1994) 4438 [hep-ph/9312276].

[14] G. Mahlon, *One loop multi-photon helicity amplitudes*, *Phys. Rev.* **D 49** (1994) 2197 [hep-ph/9311213].

[15] Z. Bern, G. Chalmers, L.J. Dixon and D.A. Kosower, *One loop N gluon amplitudes with maximal helicity violation via collinear limits*, *Phys. Rev. Lett.* **72** (1994) 2134 [hep-ph/9312333].

[16] Z. Bern, L.J. Dixon, D.C. Dunbar and D.A. Kosower, *Fusing gauge theory tree amplitudes into loop amplitudes*, *Nucl. Phys.* **B 435** (1995) 59 [hep-ph/9409265].

[17] Z. Bern, L.J. Dixon, D.C. Dunbar and D.A. Kosower, *One loop N-point gauge theory amplitudes, unitarity and collinear limits*, *Nucl. Phys.* **B 425** (1994) 217 [hep-ph/9403226].

[18] Z. Bern, L.J. Dixon, M. Perelstein and J.S. Rozowsky, *One-loop N-point helicity amplitudes in (self-dual) gravity*, *Phys. Lett.* **B 444** (1998) 273 [hep-th/9809160].

[19] D. de Florian and Z. Kunszt, *Two photons plus jet at LHC: the NNLO contribution from the gg initiated process*, *Phys. Lett.* **B 460** (1999) 184 [hep-ph/9905283].

[20] V. Del Duca, F. Maltoni, Z. Nagy and Z. Trócsányi, *QCD radiative corrections to prompt diphoton production in association with a jet at hadron colliders*, *J. High Energy Phys.* **04** (2003) 059 [hep-ph/0303012].

[21] C. Balazs, P. Nadolsky, C. Schmidt, C.P. Yuan, *Diphoton background to Higgs boson production at the LHC with soft gluon effects*, *Phys. Lett.* **B 489** (2000) 157 [hep-ph/9905551].

[22] Z. Bern, L.J. Dixon and C. Schmidt, *Isolating a light Higgs boson from the di-photon background at the LHC*, *Phys. Rev.* **D 66** (2002) 074018 [hep-ph/0206194].

[23] Z. Xu, D.H. Zhang and L. Chang, *Helicity amplitudes for multiple Bremsstrahlung in massless nonabelian gauge theories*, *Nucl. Phys.* **B 291** (1987) 392.

[24] Z. Bern, L.J. Dixon and D.A. Kosower, *Progress in one-loop QCD computations*, *Ann. Rev. Nucl. Part. Sci.* **46** (1996) 109, [hep-ph/9602280].

[25] C. Schubert, *Perturbative quantum field theory in the string-inspired formalism*, *Phys. Rept.* **355** (2001) 73 [hep-th/0101036].

[26] R. Pittau, *A simple method for multi-leg loop calculations*, *Comput. Phys. Commun.* **104** (1997) 23 [hep-ph/9607309].

[27] R. Pittau, *A simple method for multi-leg loop calculations, II. A general algorithm*, *Comput. Phys. Commun.* **111** (1998) 48 [hep-ph/9712418].

[28] S. Weinzierl, *Reduction of multi-leg loop integrals*, *Phys. Lett.* **B 450** (1999) 234 [hep-ph/9811365].

[29] A. Denner and S. Dittmaier, *Reduction of one-loop tensor 5-point integrals*, *Nucl. Phys.* **B 658** (2003) 175 [hep-ph/0212259].

[30] J.A.M. Vermaseren, *New features of FORM*, math.ph/0010025.

[31] T. Binoth, J.P. Guillet and G. Heinrich, *Reduction formalism for dimensionally regulated one-loop N-point integrals*, *Nucl. Phys.* **B 572** (2000) 361 [hep-ph/9911342].

[32] G. Heinrich and T. Binoth, *A general reduction method for one-loop N-point integrals*, *Nucl. Phys.* **89** *(Proc. Suppl.)* (2000) 246 [hep-ph/0005324].

[33] Z. Bern, L.J. Dixon and D.A. Kosower, *Dimensionally regulated one loop integrals*, *Phys. Lett.* **B 302** (1993) 299 [hep-ph/9212308].

[34] MAPLE, http://www.maplesoft.com.